Interferometry with Resonances and Flow in High-Energy Nuclear Collisions

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(Feb., 1996)

The effects of resonances and flow on the correlation function for two identical particles are described assuming chaotic sources and classical propagation of particles. Expanding to second order in relative momenta, the source sizes can be calculated directly and understood as contributions from various fluctuations in the source. Specific calculations of source size radii are given assuming Bjorken longitudinal flow with additional transverse expansion. Results are compared to recent $\pi\pi$ and KK correlation data from relativistic nuclear collisions with particular attention to the reduction in the $\pi\pi$ correlation function due to resonances and the decreasing source sizes with increasing transverse momenta of the particles.

PACS numbers:25.70.Pq

arXiv:hep-ph/9602431v2 1 Mar 1996

I. INTRODUCTION

The HBT effect based on interference of identical particles [1,2] is an important method in relativistic heavy ion collisions for extracting information about the source sizes and life times. [3] The correlations due to interference are well described for pp collisions [4] when the effect of resonances in particular the long lived ones are included. In high energy nuclear collisions we hope to extract the spatial, temporal and momentum distribution of particles in the source at decoupling or freeze-out by correcting for resonances, Coulomb effects, and possible other final state interactions. The source sizes, life times and flow effects and their dependence on collision energy, impact parameter, projectile and target mass will be crucial for determining whether a quark-gluon plasma is created. Recent data allow determination of longitudinal, outwards and sidewards source radii for pions and kaons in heavy ion collisions at CERN and Brookhaven energies. The rapidity and transverse momentum dependence of source sizes have also been measured. [5,6] The rapidity dependence of the longitudinal source size agrees with longitudinal expansion whereas decreasing transverse radii seem to indicate that also transverse expansion takes place [7,8].

Resonances are known to be abundant in relativistic heavy ion collisions and they contribute significantly to pion production by, e.g., $\sim$80% according to the Fritiof model. [9] Resonances affect the source such that it seems to have a larger life time or larger outward source size [11]. The very long lived resonances are not resolved and reduce correlations. However, the reduction is also expected if the source is partially coherent.

The purpose of this paper is to do a combined analysis of resonances and flow both longitudinal and transverse, and to calculate the nontrivial interplay. For that purpose the the correlation function for an incoherent source with plane wave propagation is extended to include resonances. By expanding in small relative momenta between the identical particles, one can extract the source radii analytically for general cylindrical symmetric sources with longitudinal and transverse flow. The effect of resonances and flow on the correlation function will be discussed and compared to recent NA44 data, which can extract sidewards, outwards and longitudinal source sizes.

II. SOURCE SIZE FLUCTUATIONS

Assuming an incoherent source and plane wave propagation, the correlation function reduces to the simple Fourier transform [12,13]

$$C(\mathbf{q},\mathbf{K}) = 1 \pm \frac{|\int d^4x\, S(x,K) e^{iq\cdot x}|^2}{|\int d^4x\, S(x,K)|^2}\,, \tag{1}$$

for two identical bosons/fermions ($\pm$) of relative and total momenta $q = k_1 - k_2$ and $K = k_1 + k_2$ respectively, when $q \ll K$. The source distribution $S(x,K) = dN/d^4xd\mathbf{K}$ describes the particle production at space-time point $x = (\mathrm{t},\mathrm{x},\mathrm{y},\mathrm{z})$ of a particle with momentum $k_1 \simeq k_2 \simeq K$.

Resonances will here be included by assuming classical propagation. The additional distance travelled by the resonance, $\Delta x = u_r\Delta\tau$, where $\Delta\tau$ is the life time of the resonance and u_r its velocity, gives an extra phase, $\Delta x\cdot q$, in Eq. (1). When the life times are exponentially distributed with decay time $\tau_r = 1/\Gamma_r$, the correlation function becomes (see also [11])

$$C(\mathbf{q},\mathbf{K}) = 1 \pm \frac{|\sum_r \int d^4x\, S_r(x,\mathbf{K}) e^{iq\cdot x}(1-iq\cdot u_r\tau_r)^{-1}|^2}{|\sum_r \int d^4x\, S_r(x,\mathbf{K})|^2}\,. \tag{2}$$

Now, x refers to the space-time production point of the resonance.

It is convenient notation to introduce the space-time integration and summation over resonances of an operator $\tilde{O}$ by the average

$$\langle \tilde{O} \rangle \equiv \frac{|\sum_r \int d^4x\, S_r(x,\mathbf{K})\, \tilde{O}|}{|\sum_r \int d^4x S_r(x,\mathbf{K})|}\,. \tag{3}$$

Likewise the fluctuation of that operator is

$$\sigma(\tilde{O}) = \langle \tilde{O}^2 \rangle - \langle \tilde{O} \rangle^2\,. \tag{4}$$

The correlation function for bosons is then

$$C(\mathbf{q},\mathbf{K}) = 1 \pm \langle e^{iq\cdot x}(1-iq\cdot u_r\tau_r)^{-1}\rangle^2\,. \tag{5}$$

Whereas the full q-dependence of the correlation function is generally quite cumbersome to calculate, the small q-dependence can be calculated directly by expanding (5) to second order in q

$$C(\mathbf{q},\mathbf{K}) = 1 \pm \left[1 - \sigma(q\cdot[x+u_r\tau_r]) - \langle [q\cdot u_r\tau_r]^2\rangle - \mathcal{O}((q\cdot x)^4)\right]\,. \tag{6}$$

The expansion is valid when $R_\mu q_\mu \ll 1$, where R_μ are the space-time extension of the system. If the source and therefore also the correlation function is approximately gaussian, then the leading term is sufficient to determine its shape. Notice that the resonance life-times contributes to the correlation function in Eq. (6) through the fluctuations in space and time, $\sigma(q\cdot[x+u_r\tau_r])$, as well as contributing by the explicit term $\langle [q\cdot u_r\tau_r]^2\rangle$.

Experimentalist often parametrize their pion and kaon correlation function by

$$C(\mathbf{q},\mathbf{K}) = 1 + \lambda \exp[-q_s^2R_s^2 - q_o^2R_o^2 - q_l^2R_l^2 - 2q_oq_lR_{ol}^2]\,, \tag{7}$$

where $\mathbf{q}_{o,s,l}$ are the usual outwards, sidewards and longitudinal projections respectively of the relative momentum in a cartesian coordinate system where $\mathbf{q}_l$ lies along the beam (or longitudinal) axis and $\mathbf{q}_s$ is perpendicular to both the beam axis and $\mathbf{K}$. The cross term with coefficient R_{ol} has recently been advocated by Heinz et al. [8]. Indications of this kind of asymmetry have been seen in experiments [6]. The prefactor λ is found to be approximately unity for kaons and about half for pions in most experiments. The reduced pion correlations may be due to "coherence" in the source or to long lived resonances as will be discussed below.

Because the momentum of the pair $\mathbf{K}$ breaks cylindrical symmetry and, with the beam line, determines the x-z plane, the average values for the z and x coordinates are generally non-vanishing in a longitudinally and transversally expanding system. Reflection symmetry in the y coordinate leads to $\langle \mathrm{y}\rangle = 0$ and we can assume that the resonance velocity $u_r = (u_t, u_x, u_y, u_z)$ has vanishing y-component, $\langle u_y\rangle = 0$. Consequently, all cross terms with R_s vanish as in (7). Since $q = (\beta_oq_o + \beta_lq_l, q_o, q_s, q_l)$, where $\beta_i = K_i/K_0$, we obtain from (6) and (7)

$$R_s^2 = \sigma(\mathrm{y}) \tag{8}$$
$$R_o^2 = \sigma(\mathrm{x} + u_x\tau_r - \beta_o(t + u_t\tau_r)) + \langle [(u_x - \beta_ou_t)\tau_r]^2\rangle \tag{9}$$
$$R_l^2 = \sigma(\mathrm{z} + u_z\tau_r - \beta_l(t + u_t\tau_r)) + \langle [(u_z - \beta_lu_t)\tau_r]^2\rangle \tag{10}$$
$$\begin{aligned} R_{ol}^2 = &\langle [x + u_x\tau_r - \beta_o(t+u_t\tau_r)][\mathrm{z} + u_z\tau_r - \beta_l(t+u_t\tau_r)]\rangle \\ &- \langle x + u_x\tau_r - \beta_o(t+u_t\tau_r)\rangle\langle \mathrm{z} + u_z\tau_r - \beta_l(t+u_t\tau_r)\rangle + \langle (u_x-\beta_ou_t)(u_z-\beta_lu_t)\tau_r^2\rangle\,. \end{aligned} \tag{11}$$

To evaluate the source radii, we need to specify the source further. Since the particle spectra have approximately thermal transverse momentum distributions in the energy and $p_\perp$ regions considered here, we will here assume that all resonances are thermally distributed locally

$$S_r(x,K) = f_r \exp[-K\cdot u(x)/T_r]\, \rho(x)\,. \tag{12}$$

Here, f_r determines the fraction of particles coming from resonance r, T_r is the "temperature" or more accurately the inverse of the $m_\perp$ slope of particles produced through that resonance, $u(x)$ is the flow velocity at a given point in space and time, and ρ is the spatial and temporal source function. For longitudinally expanding systems the proper time $\tau = \sqrt{t^2 - \mathrm{z}^2}$ and $\eta = \frac{1}{2}\ln((t+\mathrm{z})/(t-\mathrm{z}))$ are convenient variables whereby $d^4x = \tau d\tau d\eta d\mathrm{x}d\mathrm{y}$.

The source function is commonly parametrized by factorizing gaussians common for all resonances

$$\rho(\tau,\eta,\mathrm{x},\mathrm{y}) \sim \frac{1}{\tau}\exp\left[-\frac{\mathrm{x}^2+\mathrm{y}^2}{2\sigma_\perp}-\frac{(\eta-\eta_0)^2}{2\sigma_\eta}-\frac{(\tau-\langle\tau\rangle)^2}{2\sigma_\tau}\right] . \tag{13}$$

However, the detailed form will not be needed in the present analysis. Only the mean and the fluctuations in the quantities, e.g., $\langle \mathrm{x}\rangle$ and $\sigma(\mathrm{x}) = \langle \mathrm{x}^2\rangle - \langle \mathrm{x}\rangle^2$, are needed to leading order. When the experimental accurracy improves one may be able to measure finer details of correlation functions which then can determine higher moments. Whereas we can always translate our coordinate system so that $\langle \mathrm{x}\rangle = \langle \mathrm{y}\rangle = \langle \eta\rangle = 0$, the average emission or freeze-out time $\langle\tau\rangle$ does not vanish and it determines the longitudinal extension of the source. The source in Eq. (13) assumes cylindrical symmetry, i.e. $\sigma(\mathrm{x}) = \sigma(\mathrm{y})$. If one is able to determine the plane of reaction, that symmetry is broken [14] and one may determine $\sigma(\mathrm{x})$ and $\sigma(\mathrm{y})$ separately.

III. LONGITUDINAL EXPANDING SYSTEM

Let us first analyse the situation with longitudinal expansion as in the Bjorken model, i.e., $u = (\cosh\eta, 0, 0, \sinh\eta)$. Defining the mean rapidity as $Y = (y_1 + y_2)/2$, we find

$$K \cdot u = m_\perp \cosh(\eta - Y) . \tag{14}$$

The resonance velocities are approximated by their local average over all directions which is simply the local flow velocity, i.e., $u_r = u$. It is now straight forward to evaluate the source radii of (8-11). However, the gaussian in rapidity has the effect of moving the average rapidity, $\langle\eta\rangle$ (referred to as the "saddle point" in Refs. [8,7]), away from the mean rapidity, Y, of the two interfering particles. In the limit when $\sigma_\eta \gg T/m_\perp$ so that $\langle\eta\rangle \simeq Y$, the results simplify to

$$R_s^2 = \sigma(\mathrm{y}) \tag{15}$$
$$R_o^2 = \sigma(\mathrm{x}) + \beta_\perp\left[\sigma(\tau) + \sigma(\tau_r) + \langle\tau_r^2\rangle + \langle\tau+\tau_r\rangle^2 \tanh^2 Y \sigma(\eta)\right] \tag{16}$$
$$R_l^2 = \sigma(\eta)\langle\tau^2 + 2\tau\tau_r + 2\tau_r^2\rangle \cosh^{-2}(Y) \tag{17}$$
$$R_{ol}^2 = -\beta_\perp\sigma(\eta)\langle\tau+\tau_r\rangle^2 \sinh(Y)\cosh^{-2}(Y) , \tag{18}$$

where $\beta_\perp = p_\perp/m_\perp$. We observe that the source sizes are given in terms of the dispersion of the source, namely the fluctuations in spatial coordinates ($\sigma(\mathrm{x})$ and $\sigma(\mathrm{y})$), temporally ($\sigma(\tau)$), rapidity ($\sigma(\eta)$), and in resonance life times ($\sigma(\tau_r)$). For the simplified source of Eq. (13) the spatial fluctuations transversally are simply $\sigma(\mathrm{x}) = \sigma(\mathrm{y}) = \sigma_\perp$, and the temporal fluctuations are $\sigma(\tau) = \sigma_\tau$.

The fluctuation in rapidity consist not only of the spatial rapidity fluctuation in the distribution (13) but is strongly reduced by the additional rapidity dependence present in the thermal factor of (12)

$$\frac{1}{\sigma(\eta)} = \langle\frac{m_\perp}{T_r}\rangle + \frac{1}{\sigma_\eta} . \tag{19}$$

However, as discussed above, it was assumed that $\sigma_\eta \gg T/m_\perp$ in deriving Eqs. (15-18) and therefore the first term in (19) dominates. When the various resonances have different temperatures, the average over T_r will produce fluctuations in temperature, $\sigma(T_r)$, besides the average value $\langle T_r\rangle$.

TABLE I. Resonances, that decay into pions and contribute to the pion correlation function. The fractions are predicted in the Fritiof [9] and RQMD [10] models for central S+Pb collisions at energies around 200 GeV/A, midrapidity and all $m_\perp$.

Resonance	Decay	m_r	Width	$c\tau_r$	$f_r^{Fritiof}$	f_r^{RQMD}
direct π		140 MeV	-	0 fm	0.19	0.33
ρ	$\pi\pi$	770 MeV	153 MeV	1.3 fm	0.40	0.26
Δ	$N\pi$	1232 MeV	115 MeV	1.7 fm	0.06	0.12
K^*	$K\pi$	892 MeV	50 MeV	3.9 fm	0.09	0.07
Σ^*	$\Sigma\pi$	1385 MeV	36 MeV	5.5 fm	0.01	0.02
ω	$\pi\pi\pi$	783 MeV	8.4 MeV	23.4 fm	0.16	0.07
η'	$\eta\pi\pi$	958 MeV	0.24MeV	821 fm	0.02	0.02
η	$\pi\pi\pi$	549 MeV	1.1 keV	1.2 Å	0.04	0.03
K_S^0	$\pi\pi$	498 MeV	~ 0	2.7 cm	0.03	0.07
$\Sigma, \bar\Sigma$	$n\pi, \bar n\pi$	1193 MeV	~ 0	4.4 cm	0.00	0.01

The resonances contribute to fluctuations by $\sigma(\tau_r)$. When the temperatures or $m_\perp$-slopes are the same for all resonances, the fluctuation in the resonance life times is

$$\sigma(\tau_r) = \langle\tau_r^2\rangle - \langle\tau_r\rangle^2 = \sum_r f_r\tau_r^2 - (\sum_r f_r\tau_r)^2 \,, \tag{20}$$

where f_r is the fraction of pions arising from resonance r as, for example, given in Table I for the Fritiof [9] and RQMD [10] models. The resonance contributions may vary significantly with rapidity and transverse mass, i.e. the T_r's are different, and therefore the fraction at the relevant rapidity and transverse mass of the pair, $f_r(Y, m_\perp)$, should be inserted in Eq. (20).

In relativistic heavy ion collisions numerous resonances contribute to pion production as illustrated in Table I. The relative contributions from the various resonances have unfortunately not been measured very accurately and different models give a variety of results. This is illustrated by the Fritiof and RQMD models in Table I, where the feed-down to π^+ around mid-rapidity are given for central $S + Pb$ collision at 200 GeV/A corrected for detection efficiency. Note that the rescatterings in RQMD lead to a strangeness enhancement, which at mid-rapidities is mainly felt as an enhancement of the long-lived K_S^0 and to less degree of hyperons. Feed-down from $\Xi^*, \bar{\Lambda}, \phi, ...$ contribute by a few per mille each and have not been included in Table I. The feed-down to kaons is mainly from the K^* by ~50% in Fritiof and ~5% in a thermal model [15]).

Since the typical minimum relative momentum, that can be measured between pions in relativistic heavy ion collisions, is of order $q_{min} \sim 5 - 10$MeV, the resonance phase is large, $q \cdot \tau_r \gg 1$, for the long lived resonances $r = \eta, \eta', K_S^0, \Sigma, \bar{\Sigma}, ...$, and therefore the small q employed in Eq. (6) expansion is not allowed. Instead the resonance factor vanishes, $|1 - iq \cdot u_r\tau_r|^{-1} \ll 1$, for these long lived resonances and their contribution to the correlation function can be neglected. This can be taken into account by simply excluding these long lived resonances in the sum over resonances and the correlation function in (13) is reduced by a factor

$$\lambda_{res} = (1 - f_\eta - f_{\eta'} - f_{K_S^0} - f_{\Sigma,\bar{\Sigma},..})^2 \,. \tag{21}$$

The ω resonance has a life-time such that $q_{min}\tau_\omega \sim 1$ and should therefore be treated as an intermediate case.

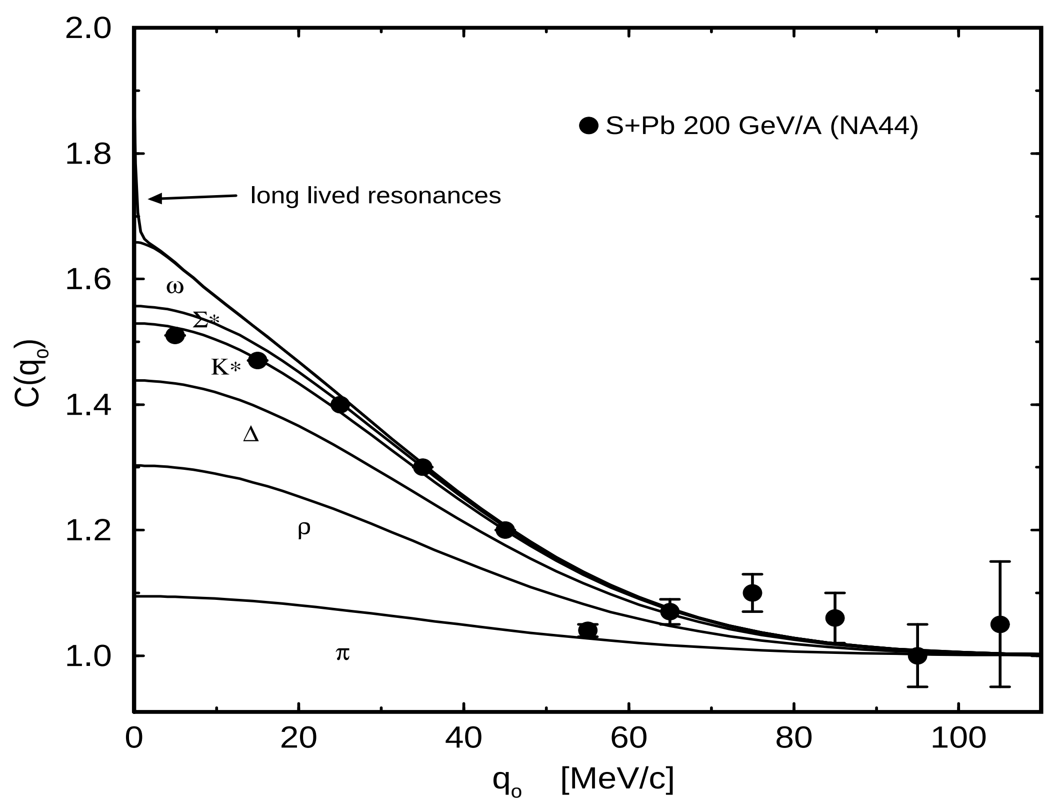

FIG. 1. Resonance contributions to the $\pi\pi$ correlation function as function of outwards relative momentum, q_o (see text). Curves include successively direct pions, ρ, Δ, K^*, Σ^*, ω and $\eta + \eta' + K_S^0 + \Sigma + \bar{\Sigma} + ...$ from RQMD (Table I). Experimental data for central S+Pb collisions at 200 GeV/A [5] is shown for comparison.

To illustrate the effects of resonances, the two-pion correlation function is shown in Fig. 1 as function of the out-ward relative momentum. The correlation functions are calculated from Eq. (5) with the gaussian source of Eq. (13), which Fourier transforms to $|\tilde{\rho}|^2 = \exp[-q_o^2\sigma_\perp]$. The transverse radius $\sqrt{\sigma_\perp} = 4$fm is assumed and the resonance fractions and life times are from RQMD as given in Table I. The curves shows the contribution from resonances by adding them successively in the order of increasing life-times. It is assumed that the source is completely incoherent and that Coulomb effects have been corrected for. The long lived resonances lead to a smaller λ value and the resonances in particular the ω makes the resulting correlation function steeper than a simple gaussian at small q_o. Many experiments actually find that the pion correlation data is fit better by steeper functions (e.g., exponentials) than gaussians.

For comparison, the $\pi^+\pi^+$ correlation function measured in S+Pb collisions at 200 GeV/A [5] is also shown in Fig. 1. The data are collected for $q_s, q_l \le q_{cut} = 20$MeV/c [5]. The cuts lead to a further reduction

$$\lambda \equiv C(q_o, q_s \le q_{cut}, q_l \le q_{cut}) - 1 = \lambda_{res}\lambda_{cut} \,, \tag{22}$$

where $\lambda_{cut} = (1 - R_s^2 q_{cut}^2/3)(1 - R_l^2 q_{cut}^2/3)$ when $q_{cut}R_{s,l} \lesssim 1$, and the quadratic expansion of Eq. (6) is valid. For the NA44 source sizes $R_s = 4.2$fm and $R_l = 4.7$fm and $q_{cut} = 20$MeV/c one finds $\lambda_{cut} = 0.87$. In the comparison with experimental data of Fig. 1, the theoretical curves include the 13% reduction from λ_{cut}. The data prefers more long lived resonances than predicted by Fritiof (see also [16]) and RQMD and there is no indication of the ω resonances. Due to strangeness enhancement of in particular K_S^0, the RQMD model describes the data better than Fritiof. In the recent Pb+Pb experiments at CERN, the small $\lambda^{\pi\pi} \simeq 0.3$ measured [17] may be explained by the larger $q_{cut} = 30$MeV and larger source sizes $R_{o,s,l} \simeq 6 - 7$fm, which lead to a smaller λ_{cut}. No λ_{cut} applies when invariant relative momenta, $q_{inv} = \sqrt{q^2}$, are employed which explains why $\lambda_{inv} = C(q_{inv} = 0) - 1$ generally are found to be larger.

The measured values for the sidewards and outwards radii are very similar in high energy nuclear collisions. For example, in *S+Pb* collisions [5] $R_o = (4.02 \pm 0.14)$fm and $R_s = (4.15 \pm 0.27)$fm so that $R_s^2 - R_s^2 = (-1.1 \pm 3.4)\text{fm}^2$. In Eq. (16) the resonances makes a difference of $\sigma(\tau_r) = 1.2\text{fm}^2$ and $\langle\tau_r^2\rangle = 3.0\text{fm}^2$ in the Fritiof model even without including ω's or the long lived resonances. The transverse velocities for the measured pions are $\beta_\perp \sim 0.6 - 0.9$. Thus the short lived resonances lead to a significant difference between $R_o^2 - R_s^2$ that is already between one and two standard deviations larger than the experimental value. This leaves very little room for additional fluctuations in source emission time, $\sigma(\tau)$, and therefore the pions must be produced in a "flash".

Due to longitudinal expansion, both the expansion time of the source $\langle\tau\rangle$ and the lifetime of the resonances τ_r as well as their fluctuations contribute to R_o, R_l and R_{ol}. The longitudinal source size scales with rapidity and transverse mass as $R_l \propto (\sqrt{m_\perp}\cosh(Y))^{-1}$ as found in [18] and according to Eq. (17) when $\sigma_\eta \ll m_\perp/T$. Experiments confirm both the rapidity dependence [6,17] and the $m_\perp$ dependence (see [5] and Fig. 2). If, for example, the thermal factor in (12) is replaced by that of free streaming ($\sim \exp(-m_\perp/T)$, see, e.g., Refs. [19]), then the resulting rapidity fluctuations would only be the second term in (19). This does not produce the observed y and $m_\perp$ dependence which indicates that some thermalization must have taken place in high energy nuclear collisions before freeze-out.

The source radii above can be Lorenzt boosted longitudinally to any frame by simply boosting the rapidity Y. In the longitudinal center-of-mass system, defined such that every pair is boosted to its c.m.s. system, the radii are obtained from the above by setting $Y = 0$. Thus the asymmetric term R_{ol} vanish in Eq. (18) to quadratic order but as pointed out in [8] terms of fourth order are non-vanishing.

Most resonances produced in relativistic nuclear collisions cannot decay into two positively or negatively charged pions. Also, a single string cannot produce two particles with the same nonvanishing charge next to each other and an anticorrelation appears. These anti-correlations in a single resonance or string are, however, completely washed out by the abundance of resonances in relativistic nuclear collisions.

IV. TRANSVERSE EXPANSION

Besides longitudinal expansion the source may also expand transversally. Transverse flow has been studied with renewed interest [7,8] since it decreases the source sizes R_o and R_s with increasing $m_\perp$, an effect that has recently been seen experimentally [5]. The analysis in [7,8] will here be extended to include resonances.

A source expanding transversally with velocity $v(x)$ has flow velocity

$$u = \gamma(v)(\cosh(\eta), v_x(x), v_y(x), \sinh(\eta)) \,, \tag{23}$$

which gives

$$K \cdot u = m_\perp\gamma(v)(\cosh(\eta - Y) - \beta_\perp v_x) \,. \tag{24}$$

Assuming the source is expanding as $v_x = vx/\sqrt{\sigma_\perp}$ and similarly for v_y one finds when $v^2 \ll 1$

$$R_s^2 = \sigma_\perp \langle \frac{1 + 2v\tau_r/\sigma_\perp^{1/2} + 2v^2\tau_r^2/\sigma_\perp}{1 + v^2 m_\perp/T_r} \rangle \tag{25}$$

$$R_o^2 = R_s^2 + (\beta_\perp^2 + v^2)\langle \tau_r^2 \rangle + \beta_\perp^2 \left[(1+v^2)\sigma(\tau_r) + \sigma(\tau) + \langle \tau + (1+v^2)\tau_r \rangle^2 \tanh^2 Y \sigma(\eta) \right] \tag{26}$$

$$R_l^2 = \sigma(\eta) \left[\langle \tau^2 \rangle + 2(1+v^2)\langle \tau\tau_r \rangle + 2(1+v^2)^2 \langle \tau_r^2 \rangle \right] \cosh^{-2}(Y) \tag{27}$$

$$R_{ol}^2 = -\beta_\perp \sigma(\eta) \langle \tau + (1+v^2)\tau_r \rangle^2 \sinh(Y)\cosh^{-2}(Y)\,. \tag{28}$$

One notices the factor $(1 + v^2 m_\perp/T)^{-1}$ which was found in earlier analyses [7,8] and which leads to smaller apparent sources at large $m_\perp$. Resonances do not change this dependence but lead to generally larger apparent sources by additional terms of order $v\tau_r/\sigma_\perp$.

The transverse flow results in smaller transverse momentum slopes or equivalently larger apparent or effective temperatures, T_{eff}. For a fixed $m_\perp$ slope, the local temperatures are "red shifted" approximately as $T = T_{eff}\sqrt{(1-v)/(1+v)}$ [20] as the transverse flow increases. Consequently, the transverse flow results in smaller source sizes at large $m_\perp$ not only due to the explicit v^2 terms in Eqs. (25-28) but also by reducing the temperature.

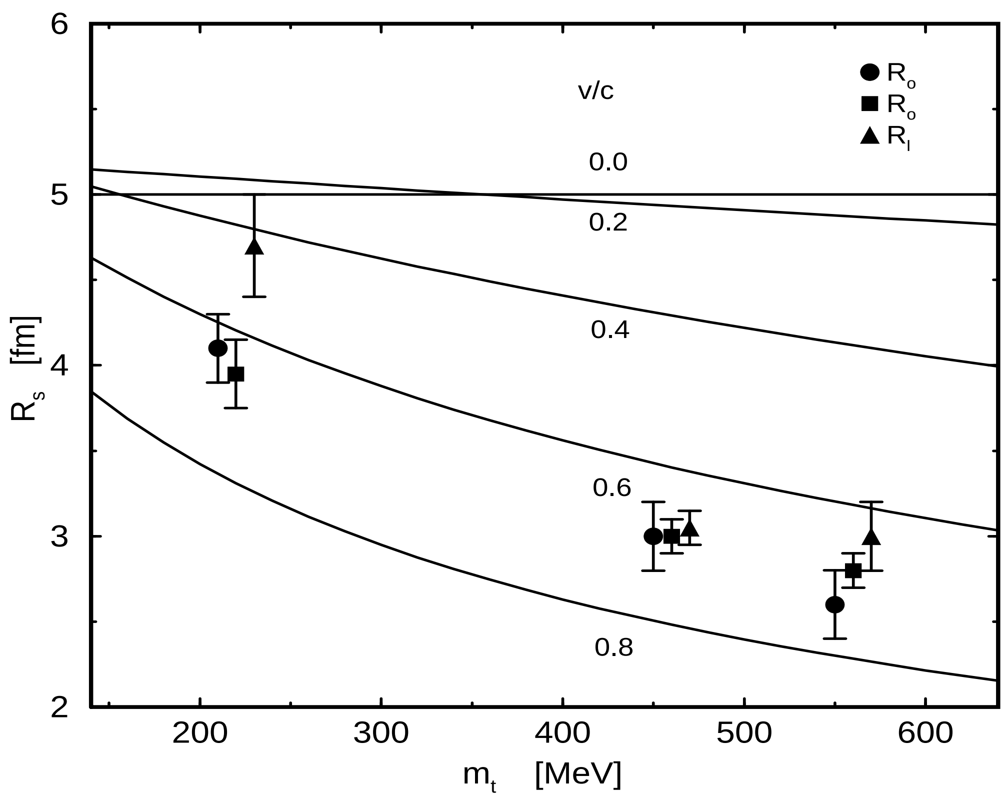

FIG. 2. Effects of transverse flow on the sidewards radius as function of transverse mass (see text). Curves correspond from top and downwards to $v/c = 0, 0.2, 0.4, 0.6, 0.8$. Experimental 200 GeV/A S+Pb data [5] on R_s, R_o, and R_l are shown (with squares, circles and diamonds respectively) for pions at two different $m_\perp$ and for kaons at $m_\perp = 550$MeV.

In Fig. 2 the $m_\perp$ dependence of the sidewards radius is shown for $T_{eff} = 170$MeV [5], $\sigma_\perp = 5$fm, and for various transverse flow velocities. As seen from Eq. (25), the scale is mainly determined by $\sigma_\perp$ whereas the curvature depends on the transverse flow velocity. The decreasing size with increasing $m_\perp$ can be reproduced with flow velocities of order $v/c \simeq 0.6 - 0.8$. These transverse velocities are smaller than those obtained by Csörgő and Lörstad. Resonance fractions from RQMD (Table I) have been employed but using fractions from Fritiof does not change the dependence on $m_\perp$ by much. The temperature is assumed to decrease with transverse flow velocity according to the red-shift formula. Including resonance fractions varying with $m_\perp$ as, e.g., in RQMD makes only minor differences in the $m_\perp$ dependence of the source sizes.

DISCUSSION

A general description of correlations and source size dependence on resonances and flow has been given. By expanding to second order in relative momenta the source radii were extracted analytically and a nontrivial interplay between the effect of resonances and flow was found. The source sizes receives contributions from fluctuations in transverse spatial directions, source life time, rapidity and resonances life times. The long lived resonances lead to a reduction in the correlation function and can explain most of the reduction in the correlations function measured experimentally, $\lambda^{\pi\pi} \simeq 0.5 - 0.6$, when the experimental cuts are included. Due to strangeness enhancement in RQMD of in particular the long lived K^0_S, it gives a smaller λ in better agreement with experiment than Fritiof. However, the similar values for $\lambda^{\pi\pi}$ in pp and $p\bar{p}$ ($\lambda \simeq 0.40$ [2][1]) and p-nucleus ($\lambda^{pA} \simeq 0.41$ [5]) and nuclear collisions do not indicate that rescattering, thermalization, strangeness enhancement or other collective effects in nuclear collisions are affecting the resonance production and λ significantly. There is no indication of the ω-resonance in the data, which is otherwise abundant in most models. From measurements of the KK correlation functions one finds $\lambda^{KK} = 0.83 \pm 0.08$ [5]), which is close to λ_{cut}, and therefore no long lived resonances decaying into kaons are required. This is predicted in the models discussed above where kaons mainly receive feed-down from the relatively short-lived K^*.

Even though resonances and cuts can explain most of the reduction in $\lambda^{\pi\pi}$ found in nuclear collisions, $\lambda^{\pi\pi} \simeq 0.5 - 0.6$ [5,21,14], it does not exclude a partially coherent pion source. However, the kaon sources measured, $\lambda^{KK}/\lambda_{cut} \sim 1$, are apparently incoherent. Furthermore, if particles are produced coherently within a coherence length, ξ_{coh}, in space and time, the reduction in the correlation function would be of order $\lambda \sim 1 - (\xi_{coh}/R)^4$. The similar λ values found in pp, $p\bar{p}$, p-nucleus and nucleus-nucleus collisions would then require that the correlation length scales approximately with the source size.

Other effects, that might explain the low $\lambda^{\pi\pi}$, are final state interactions and Coulomb repulsion. Strong interactions are generally found to be insignificant in nuclear collisions due to their short interaction range as compared to nuclear length scales. Coulomb repulsion reduce the correlation function significantly at small relative momenta, which is usually corrected for by the Gamov factor $\Gamma = \tilde{\eta}/(e^{\tilde{\eta}}-1)$, where $\tilde{\eta} = 2\pi m e^2/Q_{inv} \simeq 6\text{MeV}/Q_{inv}$ for pions. Coulomb screening effects are only of order a few percent [22] but as they reduce the Coulomb interactions, they lead to a *smaller* λ.

Experimental data seem to indicate both longitudinal and transverse flow. Longitudinal flow as in the Bjorken model seems to agree with the rapidity dependence of the longitudinal source radius. The decreasing transverse source sizes with increasing transverse mass of the particles can be explained in a simple model with transverse flow of order $v = 0.5 - 0.8c$ at the surface when resonances are included. This assumes an apparent blue-shifted temperature that is fixed by the transverse momentum slopes of particle spectra. The source sizes have been measured for two values for the pion transverse mass and one kaon transverse mass. The smaller kaon radii can thus be explained solely by transverse flow. The conventional explanation is that the smaller kaon scattering cross section results in earlier freeze-out at smaller radii than for pions. However, such a cascade picture leads to an extended particle emission time typically of the same order as the expansion time, $\sigma(\tau) \sim \langle\tau\rangle$, and not a sharp freeze-out in time as is indicated by the similarity of the sidewards and outwards radii. Also the freeze-out in hydrodynamic models at a constant temperature takes about as long as the expansion time. This is in contradiction with the very similar outwards and sidewards transverse radii measured in particular when the resonance life-times are taken into account.

Clearly, more data is crucial in order to describe the sources and determine their size, lifetime, longitudinal and transverse expansion, so that we can discriminate between these models. In particular, it would be most useful if one could measure the feed-down from resonances independently. One would then be able to predict the non-gaussian correlation function (see Fig. 1) and separate the effect of resonances from coherence, final state interactions etc. Only when resonances are properly accounted for and their effect on the correlation function, λ, the source radii and life time are understood and corrected for, can we extract the "bare" source sizes and the flow reminiscent of the earlier phase of hot and dense matter created in high energy nuclear collisions.

ACKNOWLEDGEMENTS

Discussions with Hans Bøggild, Bengt Lörstad, Heinz Sorge, John Sullivan and Axel Vischer are gratefully acknowledged.

[1]The pp and $p\bar{p}$ data are, however, at higher collision energy, $\sqrt{s} = 63\text{GeV}$, and at larger relative momentum, $q > 50\text{MeV/c}$.

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Preprint Nordita-96/15 N

hep-ph/9602431